ST. ANTHONY OF PADUA SCHOOL

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ALGEBRA – SUMMER REVIEW PACKET

Congratulations on the hard work you have put into your math studies! Algebra I is the key to the upper level math you will take in high school and college. Gaining a solid understanding of Algebraic concepts and operations is very important.

The Algebra I course you will take in eighth grade is both challenging and demanding. There will be new expectations and a more rigorous schedule. It is a high school level course which prepares you to successfully pass the Algebra Exemption Exam in May 2018.

One of the keys to a successful year of Algebra I is to continue practicing concepts already learned and utilize the skills already mastered over the summer. <u>Algebra Reteaching & Practice</u> workbook has been delivered to the school office if you have not received it yet. An answer key is provided on the school website under STUDENTS>2018-2019 SUMMER ACADEMIC ASSIGNMENTS. If your answer is incorrect, use the correct answer to work backwards and arrive at the correct answer.

The workbook is due the first day of math class. Your packet will be collected, along with the attached work which will be worth a quiz grade.

Instructions:

1. Complete workbook Chapters 1 through 3. 7.
   a. Students should **<u>not</u>** do the mixed review section of each chapter.
2. **Please complete the work on separate paper** and clearly number each problem. **Do not use a calculator for computation.** Circle each answer.
3. Use the answer key posted on the website under STUDENTS>2018-2019 Summer Academic Assignments to check answers.
   a. If the problem is incorrect, try it again by working backwards from the correct answer.
4. Please continue to practice multiplication and division facts up through 144.

Have a wonderful summer!
Ms. Jaklitsch
Middle School Math Teacher

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1 Introduction to Algebra

1–1 Variables

Objective: To simplify numerical expressions and evaluate variable expressions.

Vocabulary

Variable A symbol, such as the letter x, used to represent one or more numbers.

Value of a variable A number that a variable may represent.

Variable expression An expression, such as $x + 5$, that contains a variable.

Numerical expression, or **numeral** An expression, such as $6 + 5$, that names a particular number, called the **value of the expression.**

Simplifying an expression Replacing a numerical expression by the simplest name of its value.

Evaluating an expression Replacing each variable by a given value and simplifying the result.

Symbols $=$ (is equal to) $\neq$ (is not equal to) ab or $a \cdot b$ (multiply)

CAUTION To evaluate ab when $a = 4$ and $b = 5$, be sure to write a multiplication symbol: $ab = 4 \cdot 5$.

Example 1 Simplify: **a.** $4 + (10 - 3)$ **b.** $(20 \div 4) + (6 \div 3)$

Solution Simplify the numerical expression(s) within parentheses first.

a. $4 + \underbrace{(10 - 3)}_{7}$, $\underbrace{4 + 7}_{11}$

b. $\underbrace{(20 \div 4)}_{5} + \underbrace{(6 \div 3)}_{2}$, $\underbrace{5 + 2}_{7}$

Simplify each expression.

1. $5 + (15 - 3)$
2. $8 + (30 + 2)$
3. $16 - (0 \times 7)$
4. $13 - (4 \times 2)$
5. $(9 + 6) + 4$
6. $9 + (6 + 4)$
7. $(22 - 17) \times 6$
8. $(36 \div 9) + 16$
9. $(18 \div 3) + (10 \div 5)$
10. $(8 \times 11) - (3 \times 11)$
11. $(24 + 6) \div (30 \div 2)$
12. $(42 \div 7) \div (1 \times 3)$

Example 2 Evaluate each expression if $x = 2$ and $y = 3$.

a. xy **b.** $5xy$ **c.** $(6x) - 3$

Solution Replace x with 2. Replace y with 3. Insert the multiplication symbol(s).

a. $xy = 2 \cdot 3 = 6$ **b.** $5xy = 5 \cdot 2 \cdot 3 = 30$ **c.** $(6x) - 3 = \underbrace{6 \cdot 2}_{} - 3$
$= \underbrace{12 - 3}_{}$
$= 9$

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1-1 Variables (continued)

Evaluate each expression if $x = 3$, $y = 5$, and $z = 0$.

13. xy
14. yz
15. xz
16. $4xy$
17. $6yz$
18. $3xz$
19. $2xy$
20. $7xz$
21. $(8x) - 6$
22. $(5z) + 4$
23. $(2y) - 3$
24. $(4x) + 8$

Example 3 Evaluate each expression if $x = 6$, $y = 7$, and $z = 4$.

a. $(5x) - (2y)$ b. $6 \cdot (x + z)$ c. $\frac{(y + z)}{(y - x)}$

Solution

a. $(5x) - (2y)$

$(5 \cdot 6) - (2 \cdot 7)$ Replace x with 6 and y with 7 and insert the multiplication symbols.

$30 - 14$ Simplify the expressions within parentheses.

16 Subtract.

b. $6 \cdot (x + z)$

$6 \cdot (6 + 4)$ Replace x with 6 and z with 4.

$6 \cdot 10$ Simplify the expression within parentheses.

60 Multiply.

c. $\frac{(y + z)}{(y - x)} = \frac{7 + 4}{7 - 6}$ Replace x with 6, y with 7, and z with 4.

$= \frac{11}{1}$ Simplify the numerator and denominator.

$= 11$ Divide.

Evaluate each expression if $x = 3$, $y = 5$, and $z = 0$.

25. $(2y) - (3x)$
26. $(3y) + (5x)$
27. $(5y) - (6z)$
28. $(7z) + (4y)$
29. $5 \cdot (x - z)$
30. $4 \cdot (x + y)$
31. $\frac{(x + y)}{(y - x)}$
32. $\frac{(y + z)}{(y + z)}$

Mixed Review Exercises

Perform the indicated operations.

1. 0.2×1.3
2. $16.35 + 16.07$
3. $2.4 \div 0.6$
4. $7.3 - 5.6$
5. $106.4 + 7.8$
6. $6.72 - 3.9$
7. 50.26×1.2
8. $64 \div 0.2$
9. $\frac{3}{4} + \frac{1}{10}$
10. $\frac{3}{8} \times \frac{16}{27}$
11. $\frac{1}{4} \div \frac{3}{8}$
12. $\frac{9}{16} - \frac{3}{8}$
13. $\frac{2}{5} \times \frac{10}{3}$
14. $\frac{7}{20} \times \frac{5}{21}$
15. $\frac{5}{8} - \frac{1}{2}$
16. $\frac{6}{7} \div \frac{3}{14}$

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1–2 *Grouping Symbols*

Objective: To simplify expressions with and without grouping symbols.

Vocabulary/Symbols

Grouping symbol A symbol used to enclose an expression that should be simplified first. Multiplication symbols are often left out of expressions with grouping symbols. For example:

Parentheses	Brackets	Fraction Bar
$6(5-3)=6\cdot 2$	$6[5-3]=6\cdot 2$	$\frac{10+6}{9-5}=\frac{16}{4}$

CAUTION When there are no grouping symbols, simplify in the following order:
1. Do all multiplications and divisions in order from left to right.
2. Do all additions and subtractions in order from left to right.

Example 1 Simplify: **a.** $8(7-2)$ **b.** $8(7)-2$

Solution **a.** $8(7-2)$ The parentheses tell you to simplify $7-2$ first.

$8(5)$ $8(5)$ means $8\cdot 5$.

40

b. $8(7)-2$ Do the multiplication $8\cdot 7$ first.

$56-2$ Then subtract 2.

54

Simplify each expression.

1. a. $9(6-1)$
b. $9(6)-1$

2. a. $12(5-3)$
b. $12(5)-3$

3. a. $6+4\cdot 5$
b. $(6+4)\cdot 5$

4. a. $8+5\cdot 2$
b. $(8+5)\cdot 2$

5. a. $9-6\div 3$
b. $(9-6)\div 3$

6. a. $12+8\div 4$
b. $(12+8)\div 4$

Example 2 Simplify: **a.** $\frac{15+3}{9-3}$ **b.** $\frac{8\cdot 5+2}{2(8-5)}$

Solution **a.** $\frac{15+3}{9-3}=\frac{18}{6}$ Simplify the numerator and denominator first.

$=3$ Then divide by 6.

b. $\frac{8\cdot 5+2}{2(8-5)}=\frac{40+2}{2(3)}$ Start to simplify the numerator and denominator.

$=\frac{42}{6}$ Further simplify the numerator and denominator.

$=7$ Then divide by 6.

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1-2 Grouping Symbols *(continued)*

Simplify each expression.

7. $\frac{6 + 9}{7 - 2}$ **8.** $\frac{11 - 3}{2 + 6}$ **9.** $\frac{15 + 3 \cdot 3}{7 + 5}$ **10.** $\frac{8 \cdot 5 - 4}{3(5 - 3)}$

11. $\frac{3(11 - 7)}{2 \cdot 5 - 4}$ **12.** $\frac{6(13 - 3)}{2 \cdot 5 + 2}$ **13.** $\frac{6 \cdot 3 + 2 \cdot 7}{2(9 - 5)}$ **14.** $\frac{7 \cdot 4 - 2 \cdot 5}{3(5 - 3)}$

Example 3 Evaluate each expression if $a = 6$, $b = 2$, $c = 3$, and $d = 0$.

a. $a(b + c)$ **b.** $\frac{8(c + d)}{a - b}$

Solution

a. $a(b + c) = 6(2 + 3)$ Replace a with 6, b with 2, and c with 3.
$= 6(5)$ Simplify the expression within parentheses.
$= 30$ Multiply.

b. $\frac{8(c + d)}{a - b} = \frac{8(3 + 0)}{6 - 2}$ Replace the variables with their given values.
$= \frac{8(3)}{4}$ Simplify the numerator and denominator.
$= \frac{24}{4}$ Divide.
$= 6$

Evaluate each expression if $x = 2$, $y = 4$, $z = 6$, and $b = 5$.

15. a. $2x + 5$ **b.** $2(x + 5)$
16. a. $5y - 1$ **b.** $5(y - 1)$
17. a. $16 - 3b$ **b.** $(16 - 3)b$
18. a. $3z + 4$ **b.** $3(z + 4)$

19. a. $bx + y$ **b.** $b(x + y)$
20. a. $xz - b$ **b.** $x(z - b)$
21. a. $2xy + z$ **b.** $2(xy + z)$
22. a. $6xyz - b$ **b.** $6x(yz - b)$

23. $5(4y - 3x)$ **24.** $6z - 2xy$ **25.** $xyz - 5$ **26.** $x(y \cdot y + z)$

27. $\frac{9x + z}{x + z}$ **28.** $\frac{8x - z}{z - b}$ **29.** $\frac{9y - z}{5(b - y)}$ **30.** $\frac{2(x + y)}{x + y}$

Mixed Review Exercises

Simplify.

1. $(12 - 6) \div 3$ **2.** $20 \cdot 8 + 18 \cdot 2$ **3.** $5 \times (25 - 7)$

4. $9 + 15 \div 3$ **5.** $(25 + 3) \div (8 \div 2)$ **6.** $(7 + 5) \cdot (8 - 2)$

Evaluate each expression if $a = 2$, $b = 3$, and $c = 4$.

7. $5ab$ **8.** bc **9.** $(2c) - 3$

10. $\frac{a + c}{c - a}$ **11.** $(7a) - (4b)$ **12.** $6a$

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1–3 Equations

Objective: To find solution sets of equations over a given domain.

Vocabulary

Equation An equation is formed by placing an equals sign between two numerical or variable expressions. Examples: $2 + 3 = 5$, $x - 1 = 7$

Open sentence A sentence containing variables.

Domain of a variable The given set of numbers a variable may represent.

Solution, or root, of an equation A value of a variable that turns an open sentence into a true statement. For example, 8 is the solution of the equation $x - 1 = 7$.

Solution set of an equation The set of all the solutions of an equation.

Symbol

$\in$ (is an element of, or belongs to)

Example 1 The domain of x is $\{0, 1, 2\}$.
Is the equation $3x - 1 = 5$ true when $x = 0$? when $x = 1$? when $x = 2$?

Solution Replace x in turn by 0, 1, and 2.

x	$3x - 1 = 5$	
0	$3 \cdot 0 - 1 = 5$	False
1	$3 \cdot 1 - 1 = 5$	False
2	$3 \cdot 2 - 1 = 5$	**True**

Example 2 Read: **a.** $y \in \{1, 2, 3\}$ **b.** $x \in \{0, 2, 4, 6\}$

Solution **a.** y belongs to the set whose members are 1, 2, and 3.

b. x belongs to the set whose members are 0, 2, 4, and 6.

Example 3 Solve $y(3 - y) = 2$ if $y \in \{0, 1, 2, 3\}$.

Solution Replace y in turn with 0, 1, 2, and 3.

y	$y(3 - y) = 2$	
0	$0(3 - 0) = 2$	False
1	$1(3 - 1) = 2$	**True**
2	$2(3 - 2) = 2$	**True**
3	$3(3 - 3) = 2$	False

The solutions are 1 and 2.

The solution set is $\{1, 2\}$.

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1–3 *Equations* *(continued)*

Solve each equation if $x \in \{0, 1, 2, 3, 4, 5\}$.

1. $x + 3 = 7$	2. $5 + x = 9$	3. $x - 3 = 2$
4. $x - 1 = 3$	5. $6 - x = 1$	6. $5 - x = 2$
7. $x + 3 = 3$	8. $x + 2 = 5$	9. $2x = 8$
10. $3x = 12$	11. $4x = 0$	12. $5x = 25$
13. $x \div 2 = 1$	14. $x \div 1 = 3$	15. $\frac{1}{2}x = 1$
16. $4x = 16$	17. $x \cdot x = 4$	18. $5x = 5$
19. $x \cdot x = 9$	20. $x \cdot x = 16$	21. $3x + 7 = 19$
22. $5x - 2 = 13$	23. $x(5 - x) = 6$	24. $x(4 - x) = 3$

Example 4 Solve over the domain $\{2, 4, 6\}$.
Three more than twice a number is 11. What is the number?

Solution Use mental math to see which members of the given domain are solutions.

Number	Three more than twice a number is 11.	
2	Three more than twice 2 is 11.	False
4	Three more than twice 4 is 11.	**True**
6	Three more than twice 6 is 11.	False

The number is 4.

Solve each problem over the domain $\{2, 3, 4, 5\}$.

25. Eleven more than a number is 15. What is the number?
26. Four times a number is 16. What is the number?
27. A number divided by one is 5. What is the number?
28. Two less than a number is 3. What is the number?
29. One less than twice a number is 9. What is the number?
30. One more than twice a number is 7. What is the number?

Mixed Review Exercises

Simplify.

1. $9 \cdot 8 + 9 \cdot 12$	2. $8 + (12 \div 2)$	3. $(16 - 7) \div 3$
4. $(2 + 3 \cdot 4) \div 7$	5. $15 - 9 \div 3 \div 3$	6. $35 \div 7 \div (3 + 2)$

Evaluate if $a = 2, x = 3, y = 5$, and $z = 6$.

7. $2x + 3y$	8. $8 \cdot (z - x)$	9. $3(ax + 2)$
10. $3xz + 2y$	11. $axz \div (y + 1)$	12. $4a \div (x + 1)$

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1–4 Translating Words into Symbols

Objective: To translate phrases into variable expressions.

	Phrase	Variable Expression
Addition	The *sum* of 6 and x A number *increased* by 5 3 *more than* a number	$6 + x$ $n + 5$ $n + 3$
Subtraction	The *difference* between a number and 7 A number *decreased* by 6 3 *less than* a number 9 *minus* a number	$x - 7$ $x - 6$ $n - 3$ $9 - n$
Multiplication	The *product* of 6 and a number Five *times* a number One half *of* a number	$6n$ $5n$ $\frac{1}{2}x$
Division	The *quotient of* a number and 4 A number *divided* by 8	$\frac{n}{4}$ $\frac{n}{8}$

CAUTION The phrase "6 less than x" is translated $x - 6$ and *not* $6 - x$.
The phrase "6 more than x" can be translated as either $6 + x$ or $x + 6$.

Example 1 Translate each phrase into a variable expression.

a. Five less than half of x

b. One half the difference between x and 5

Solution

a. Half of x: $\frac{1}{2}x$

Five less than half of x: $\frac{1}{2}x - 5$

b. The difference between x and 5: $(x - 5)$

One half the difference between x and 5: $\frac{1}{2}(x - 5)$

Translate each phrase into a variable expression. Use n for the variable.

1. Five more than a number
2. The product of 7 and a number
3. A number divided by 4
4. Seven less than a number
5. The sum of 3 and a number
6. A number decreased by 8
7. The quotient of a number and 3
8. The difference between a number and 6
9. Nine times a number
10. The difference between 9 and a number
11. Four more than half a number
12. Two less than 4 times a number
13. Ten less than one half a number
14. The quotient of 3 and a number
15. Three plus the product of a number and 5
16. The difference between 3 times a number and 6
17. Four more than three times a number
18. Nine increased by twice a number
19. Eight times the sum of a number and 2
20. Seven times the difference of a number and 5

1-4 *Translating Words into Symbols* *(continued)*

Example 2	Complete the statement with a variable expression.	**Solution**
	Lila is 3 in. shorter than Dale. If Dale's height is x in., then Lila's height is _?_ in.	$x - 3$

Complete each statement with a variable expression.

21. Lisa is 2 cm taller than Fred.
If Fred's height is f cm, then Lisa's height is _?_ cm.

22. Arnie is 7 in. shorter than Rick.
If Rick's height is r in., then Arnie's height is _?_ in.

23. Shawn has $6 more than Maria.
If Maria has m dollars, then Shawn has _?_ dollars.

24. Barb has twice as much money as Carlos.
If Carlos has c dollars, then Barb has _?_ dollars.

Example 3	Complete each statement with a variable expression.	**Solution**
	a. The sum of two numbers is 15. If one number is x, then the other number is _?_.	**a.** $15 - x$
	b. The product of two numbers is 24. If one number is y, then the other number is _?_.	**b.** $24 \div y$, or $\frac{24}{y}$

Complete each statement with a variable expression.

25. The sum of two numbers is 9.
If one number is n, then the other number is _?_.

26. The product of two numbers is 15.
If one number is y, then the other number is _?_.

27. The sum of two numbers is 12.
If one number is x, then the other number is _?_.

28. The product of two numbers is 20.
If one number is w, then the other number is _?_.

Mixed Review Exercises

Evaluate if $t = 3$, $x = 4$, $y = 5$, and $z = 6$.

1. $5x - 2$ **2.** $2 + xyz$ **3.** $(3x - 1) \cdot 2$

4. $2x + 3y - t$ **5.** $tyz - 1$ **6.** $xy + t + z$

Solve if $x \in \{0, 1, 2, 3, 4\}$.

7. $x + 3 = 7$ **8.** $3x = 6$ **9.** $5x = 5$ **10.** $3x = 0$

11. $2x - 1 = 5$ **12.** $5 = 2x + 1$ **13.** $2x = x + 1$ **14.** $x \div 4 = 1$

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1–5 Translating Sentences into Equations

Objective: To translate word sentences into equations.

Example 1	Twice the sum of a number and 3 is twelve.
Translation	$2 \cdot (n + 3) = 12$

Example 2	The sum of one half of the number x and 10 is 24.
Translation	$\frac{1}{2}x + 10 = 24$

Match the sentence in the first column with the corresponding equation in the second column.

1. Three more than twice a number is nine.
2. Two less than three times a number is nine.
3. Three times the number which is two less than x is nine.
4. Two times the number which is three less than x is nine.
5. Two times the quantity three more than x is nine.
6. Three less than the product of two and x is nine.
7. Two decreased by three times a number is nine.
8. Three times the quantity two decreased by x is nine.

a. $2 - 3x = 9$
b. $3(x - 2) = 9$
c. $2x + 3 = 9$
d. $2(x + 3) = 9$
e. $3(2 - x) = 9$
f. $2(x - 3) = 9$
g. $2x - 3 = 9$
h. $3x - 2 = 9$

Translate each sentence into an equation.

9. One half of a number is four.
10. Three more than a number is eight.
11. Six less than a number is nine.
12. Two less than three times a number is eleven.
13. Twice a number is 12 more than five times the number.
14. The number x is seven more than one fourth of itself.
15. Five less than twice a number is 15.
16. Two times the quantity x minus 1 is 12.
17. Eleven more than twice x is five less than x.
18. Nine times x is twice the sum of x and five.

Vocabulary

Formulas Equations that state rules about relationships. Examples:

$A = lw$	**Area of rectangle** = length of rectangle × width of rectangle
$P = 2l + 2w$	**Perimeter of rectangle** = (2 × length) + (2 × width)
$D = rt$	**Distance traveled** = rate × time traveled
$C = np$	**Cost** = number of items × price per item

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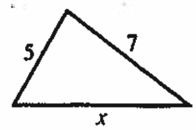
1–5 *Translating Sentences Into Equations* *(continued)*

Example 3 Use the figure and the information below it to write an equation involving x.

Solution Perimeter = the sum of the lengths of the sides.

$20 = 5 + 7 + x$

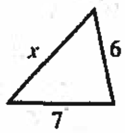
$20 = 12 + x$



Perimeter = 20

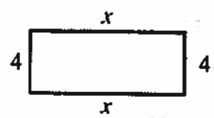
Use the figure and the information below to write an equation involving x.

19.



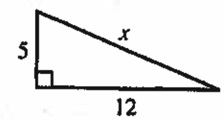
Perimeter = 21

20.



Perimeter = 28

21.



Perimeter = 30

Example 4 **a.** Choose a variable to represent the number described in parentheses.
b. Write an equation that represents the given information.

The distance traveled in 4 h of driving was 260 km. (Hourly rate)

Solution 1 **a.** Let r = the hourly rate
b. Rate × time = distance
$r \cdot 4 = 260$,
or $4r = 260$.

Solution 2 **a.** Let r = the hourly rate
b. Since the hourly rate is the number of km traveled in one hour, $r = \frac{260}{4}$.

In Exercises 22–24,
a. Choose a variable to represent the number described in parentheses.
b. Write an equation that represents the given information.

22. The distance traveled in 3 h of driving was 210 km. (Hourly rate)

23. A train traveled at 66 km/h for 4 h. (Distance traveled)

24. A driver averaged 60 km/h while driving 300 km. (Time)

Mixed Review Exercises

Solve if $x \in \{0, 1, 2, 3, 4, 5, 6\}$.

1. $3 + x = 8$ **2.** $4 = x - 2$ **3.** $4x = 20$ **4.** $2 = x \div 3$

5. $2x + 1 = 7$ **6.** $3x = x + 4$ **7.** $x + 3 = 2x$ **8.** $2x = x \cdot 2$

Translate each phrase into a variable expression.

9. A number increased by 6 **10.** The quotient of x and 2

11. The product of 9 and a number **12.** Twice the sum of a number and 3

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1–6 Translating Problems into Equations

Objective: To translate word problems into equations.

Step 1 **Read the problem carefully.**

Step 2 **Choose a variable and represent the unknowns.**

Step 3 **Reread the problem and write an equation.**

Example 1 Translate the problem into an equation. Do not solve the equation.

(1) Rudy has \$15 more than Franco.
(2) Together they have \$65.
How much money does each have?

Solution Use the three steps shown above.

Step 1 The unknowns are the amounts of money Rudy and Franco have. Each numbered sentence gives you a fact.

Step 2 Choose a variable for one unknown: Let f = Franco's amount.
Use f and fact (1): Then $f + 15$ = Rudy's amount

Step 3 Use f and fact (2) to write an equation: $f + (f + 15) = 65$.

Translate each problem into an equation. Do not solve the equation.

1. (1) Shelley has \$10 more than Michele.
   (2) Together they have \$50.
   How much money does each have?
2. (1) Bart has twice as much money as Elmer.
   (2) Together they have \$75.
   How much money does each have?
3. (1) Sandy sold four more cars than Michael.
   (2) Together they sold a total of 14 cars.
   How many cars did each sell?
4. (1) Joseph worked 8 h less than Lois.
   (2) Together they worked 72 h.
   How many hours did each work?
5. (1) Gloria spent \$2 more for dinner than Lucy.
   (2) Together they spent \$26.
   How much did each spend?
6. (1) Nancy jogged 3 mi less than Pat.
   (2) Together they jogged 7 mi.
   How far did each jog?
7. (1) Ray has three times as many compact discs as Mia.
   (2) Together they have 20 compact discs.
   How many compact discs does each have?

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1–6 *Translating Problems Into Equations* *(continued)*

Example 2 Translate the problem into an equation. Do not solve the equation.

(1) A 12 ft piece of pipe is cut into two pieces.
(2) One piece is 2 ft longer than the other.
What are the lengths of the pieces?

Solution Use the three steps shown on page 11.
Make a sketch to help you understand the problem.

Step 1 The unknowns are the lengths of the pieces.
Sentences (1) and (2) each give a fact.

Step 2 Choose a variable for one unknown:
Let x = the shorter piece.

Use x and sentence (2):
Then $x + 2$ = the longer piece.

Step 3 Use fact (1) to write an equation: $x + (x + 2) = 12.$

Translate each problem into an equation. Draw a sketch to help you.

8. (1) A 25 ft piece of cable is cut into two pieces.
(2) One piece is 5 ft longer than the other.
What are the lengths of the pieces?

9. (1) Kim walked twice as far as Paula.
(2) The sum of the distances they walked is 12 km.
How far did each walk?

10. (1) Miguel drove 20 km more than Jorge.
(2) Miguel drove three times as far as Jorge.
How far did each drive?

11. (1) A 16 ft piece of wood is cut into 2 pieces.
(2) One piece is 4 ft shorter than the other.
How long is each piece?

Mixed Review Exercises

Solve if $x \in \{0, 1, 2, 3, 4, 5, 6\}$.

1. $2x + 1 = 7$
2. $x \div 3 = 2$
3. $15 = 15x$
4. $5 = 3x - 1$
5. $3 + 3x = 15$
6. $5 = 2x - 7$
7. $4x = x + 3$
8. $x \cdot x = 9$

Translate each phrase into a variable expression.

9. One half of a number
10. Two more than five times a number
11. Five less than three times a number
12. Four more than one third a number

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1–7 A Problem Solving Plan

Objective: To use the five-step plan to solve word problems over a given domain.

Plan for Solving a Word Problem

Step 1 Read the problem carefully.
Find what the unknowns are and what the facts are.
Making a sketch may help.

Step 2 Choose a variable.
Use it with the given facts to represent the unknowns.

Step 3 Reread the problem.
Write an equation that represents relationships among the numbers.

Step 4 Solve the equation and find the unknowns.

Step 5 Check your results with the words of the problem.
Give the answer.

Symbol $\stackrel{?}{=}$ (Are they equal?)

Example Solve using the five-step plan. Write out each step. A choice of possible numbers for one unknown is given.

One number is 12 more than another number. Their sum is 108.
Find the numbers.
Choices for the smaller number: 40, 48, 56

Solution

Step 1 The unknowns are the two numbers.

Step 2 Let n = the smaller number. Then $n + 12$ = the larger number.

Step 3 $n + (n + 12) = 108$.

Step 4 Replace n in turn by 40, 48, and 56.

n	$n + (n + 12) = 108$	
40	$40 + (40 + 12) \stackrel{?}{=} 108$	False
48	$48 + (48 + 12) \stackrel{?}{=} 108$	**True**
56	$56 + (56 + 12) \stackrel{?}{=} 108$	False

Smaller number: $n = 48$
Larger number: $n + 12 = 60$

Step 5 Check the results of Step 4 with the words of the problem.

One number is 12 more than another number. $60 - 48 \stackrel{?}{=} 12$
$12 = 12 \checkmark$

Their sum is 108. $48 + 60 \stackrel{?}{=} 108$
$108 = 108 \checkmark$

The numbers are 48 and 60.

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1–7 *A Problem Solving Plan* *(continued)*

Solve using the five-step plan. Write out each step. A choice of possible numbers for one unknown is given.

1. In the chorus there are 12 more girls than boys. There are 32 students in the chorus. How many are boys? How many are girls?
   Choices for the number of boys: 8, 10, 12
2. There were twice as many four-door cars as two-door cars in the parking lot. There were 36 cars in the lot. How many had four doors? How many had two doors?
   Choices for the number of four-door cars: 12, 18, 24
3. One number is 52 more than another number. Their sum is 176. Find the numbers.
   Choices for the smaller number: 62, 72, 82
4. One number is three times another number. Their sum is 180. Find the numbers.
   Choices for the smaller number: 25, 35, 45
5. Art has three times as much money as Flora. Together they have $180. How much money does each person have?
   Choices for Flora's amount: 45, 60, 75
6. In a class election, the winner had 18 more votes than the loser. If 104 class members voted, how many votes did the winner receive? How many votes did the loser receive?
   Choices for the loser's votes: 29, 43, 61
7. One number is twice another number. The larger number is also 32 more than the smaller number. Find the numbers.
   Choices for the smaller number: 32, 40, 48
8. The house is 12 years older than the garage. The house is also three times as old as the garage. How old is each building?
   Choices for the garage's age: 3, 6, 9
9. Lupita has five times as much money as David. Lupita also has $20 more than David. How much money does each person have?
   Choices for David's amount: 4, 5, 6

Mixed Review Exercises

Simplify.

1. $\dfrac{6 \cdot 3 + 6 \cdot 7}{13 - 5 + 12}$
2. $(27 - 3 + 6 \div 2) \div (5 + 4)$
3. $50 - (16 \div 4 + 2)$
4. $8 \cdot 15 + 15 \cdot 2$

Translate each sentence into an equation.

5. Five times a number is 30.
6. Five is three less than a number.
7. One more than twice a number is 15.
8. One fifth of a number is four.

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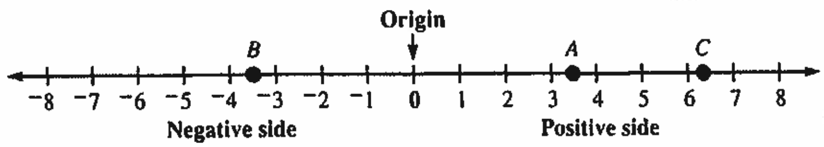
1–8 *Number Lines*

Objective: To graph real numbers on a number line and to compare real numbers.

Symbols ... (and so on) $<$ (is less than) $>$ (is greater than)

Vocabulary



Graph of a number C is the graph of $6\frac{1}{3}$.

Coordinate of a point $6\frac{1}{3}$ is the coordinate of C.

Positive integers {1, 2, 3, ...}

Positive number Example: 3.5

Negative integers $\{^{-}1, {}^{-}2, {}^{-}3, \ldots\}$

Negative number Example: $^{-}3.5$

Integers $\{\ldots, {}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3, \ldots\}$

Whole numbers {0, 1, 2, 3, ...}

Real number Any number that is either positive, negative, or zero.

Example 1 Write a number to represent each situation.

a. A deposit of \$5: 5
A withdrawal of \$5: $^{-}5$

b. 6.2 km east: 6.2
6.2 km west: $^{-}6.2$

c. 10° above freezing: 10
10° below freezing: $^{-}10$

Write a number to represent each situation. Then write the opposite of that situation and write a number to represent it.

1. Five wins
2. Six floors up
3. A gain of eight yards
4. Three points under par
5. 150 m above sea level
6. 22 km east
7. Six bonus points
8. Ten steps down
9. Four steps to the right
10. 7° above freezing (0°C)
11. A gain of two pounds
12. Latitude of 20° north
13. A bank deposit of \$50
14. A loss of \$24

Example 2 Translate each statement into symbols.

a. Three is greater than negative five

b. Negative six is less than negative two.

Solution a. $3 > {}^{-}5$ b. $^{-}6 < {}^{-}2$

Translate each statement into symbols.

15. Four is greater than negative two.
16. Negative ten is less than negative two.
17. Negative four is greater than negative six.
18. Seven is less than nine.
19. Two is less than two and two tenths.
20. Negative five tenths is less than zero.
21. Negative 12 is less than three.
22. One half is greater than one fifth.

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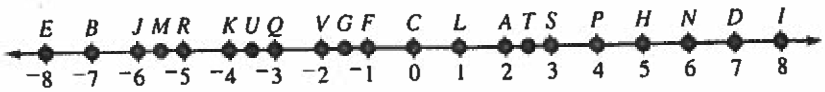
1-8 Number Lines *(continued)*

Example 3 List the letters of the points on the number line whose coordinates are given: $^{-}4$, 2.5, 0



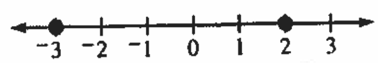
Solution *K, T, C*

List the letters of the points whose coordinates are given. Use the number line in Example 3.

23. $^{-}6$, 3 **24.** $^{-}3$, 5 **25.** 0, $^{-}7$ **26.** $^{-}5$, 1

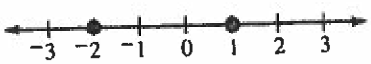
27. 2, $^{-}2$, $^{-}3\frac{1}{2}$ **28.** 0, 1, $^{-}1\frac{1}{2}$ **29.** $^{-}3$, $^{-}5\frac{1}{2}$, $^{-}2$ **30.** $^{-}7$, $^{-}5$, $2\frac{1}{2}$

Example 4 State two inequalities, one with > and one with < , relating the coordinates of the points indicated by the heavy dots.



Solution On the number line, a number to the right of another is the greater. $2 > {}^{-}3$, ${}^{-}3 < 2$

State two inequalities, one with > and one with < , relating the coordinates of the points indicated by the heavy dots.

31.


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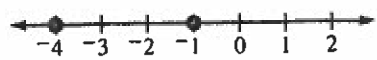

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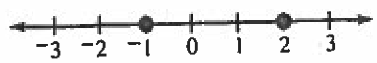

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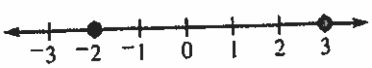

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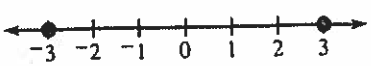

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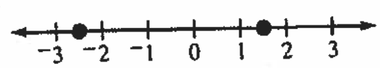

Mixed Review Exercises

Evaluate if $a = 2$, $b = 4$, $c = 3$, $x = 5$, and $y = 6$.

1. $2ab - 3x$ **2.** $3a(y - b)$ **3.** $x(2y \div 6)$

4. $\frac{1}{2}(5x - 3)$ **5.** $4y - (5b \div x)$ **6.** $\left(\frac{1}{3}ab\right) + (5 - x) \cdot c$

Translate each sentence into an equation.

7. Nine more than a number is five. **8.** The product of a number and 9 is 27.

9. One third of a number is four. **10.** A number decreased by 18 is 30.

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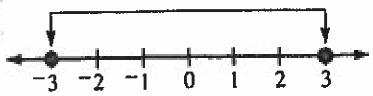
1-9 Opposites and Absolute Values

Objective: To use opposites and absolute values

Vocabulary

Opposite of a number Each number in a pair such as 3 and $^{-}3$ is called the opposite of the other number. The opposite of 0 is 0.



Absolute value of a number The distance between the graph of the number and 0 on a number line. For example, the absolute value of both 3 and $^{-}3$ is the positive number 3.

Symbols

The *opposite of a* is written $-a$. Since -3 and $^{-}3$ name the same number, you can use the lowered minus sign to write negative numbers from now on.

$|a|$ means *the absolute value of a.*

Examples: $|3| = 3$ $|-3| = 3$ $|0| = 0$

CAUTION $-a$, read "the opposite of a," is not necessarily a negative number. For example, if $a = -3$, then $-a = -(-3) = 3$.

Example 1	Simplify:	**a.** $-(5 + 4)$	**b.** $-(-2.1)$	**c.** $-(8 - 5)$
Solution		**a.** $-(9) = -9$	**b.** 2.1	**c.** $-(3) = -3$

Simplify.

1. $-(-3)$ **2.** $-(9)$ **3.** $-(2 + 6)$ **4.** $-(6 - 2)$ **5.** $-(5 + 9)$

6. $-(6 - 6)$ **7.** $-(6 + 6)$ **8.** $-(-3.5)$ **9.** $-(2.9)$ **10.** $-(7 - 3)$

Example 2	Simplify:	**a.** $|-2| + 3$	**b.** $|-3| - |-2|$
Solution		**a.** $|-2| + 3 = 2 + 3 = 5$	**b.** $|-3| - |-2| = 3 - 2 = 1$

Simplify.

11. $6 + |-4|$ **12.** $|-9| + 2$ **13.** $|-2| + |-6|$

14. $|-1.5| + |1.5|$ **15.** $|-0.2| + |-1.8|$ **16.** $\left|-\frac{1}{4}\right| + 0$

17. $\left|-\frac{3}{4}\right| + \left|\frac{1}{4}\right|$ **18.** $|-6| + |-0.5|$ **19.** $|8| - |-6|$

20. $\left|-\frac{4}{5}\right| - \left|\frac{1}{5}\right|$ **21.** $|-8| - |8|$ **22.** $|7| - |-7|$

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1–9 *Opposites and Absolute Values* *(continued)*

Example 3 Use one of the symbols >, <, or = to make a true statement.

a. $-8 \underline{?} -(-8)$ b. $|-6| \underline{?} 6$

Solution a. $-8 \underline{?} 8$ b. $6 \underline{?} 6$

$-8 < 8$ $6 = 6$

Use one of the symbols >, <, or = to make a true statement.

23. $-(-3) \underline{?} -3$
24. $-2 \underline{?} -(-2)$
25. $-(-4) \underline{?} |-5|$
26. $|-8| \underline{?} |8|$
27. $|-12| \underline{?} |-8|$
28. $|-6| \underline{?} |-10|$
29. $-|-3| \underline{?} -3$
30. $-2 \underline{?} -|-2|$
31. $-|-9| \underline{?} |-7|$

Example 4 Solve each equation over the set of real numbers. If there is no solution, explain why there is none.

a. $|x| = 2$ b. $|n| = -5$

Solution a. Both -2 and 2 are 2 units from the origin, so the replacements for x that make $|x| = 2$ true are -2 and 2. Therefore the solution set is $\{-2, 2\}$.

b. The absolute value of a number is never negative. Therefore there is no solution.

Solve each equation over the set of real numbers. If there is no solution, explain why there is none.

32. $|n| = 1$
33. $|x| = 5$
34. $|t| = \frac{1}{4}$
35. $|z| = 0.5$
36. $|a| = -3$
37. $|m| = -6$
38. $|-q| = 7$
39. $|-c| = 2$
40. $|-w| = -4$

Mixed Review Exercises

Simplify.

1. $7 - (3 + 2)$
2. $7 - 3 + 2$
3. $12 \div (2 + 4)$
4. $(7 - 2) \cdot (5 - 3)$
5. $7 - 3 \cdot 2$
6. $18 - 10 \div (3 + 2)$

Write a number to represent each situation. Then write the opposite of that number.

7. Four steps up
8. A bank deposit of $80
9. A loss of $100
10. Nine kilometers east

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2 Working with Real Numbers

2–1 *Basic Assumptions*

Objective: To use number properties to simplify expressions.

Vocabulary

Unique One and only one

Terms When a and b are added, a and b are called terms.

Factors When a and b are multiplied, a and b are called factors.

Properties of Real Numbers	Addition	Multiplication
Closure Properties The sum and product of any two real numbers are also real numbers and they are unique.	$2 + 3 = 5$ and only 5	$2 \cdot 3 = 6$ and only 6
Commutative Properties The order in which you add or multiply any two real numbers does not affect the result.	$3 + 5 = 5 + 3$	$3 \cdot 5 = 5 \cdot 3$
Associative Properties When you add or multiply any three real numbers, the grouping (or association) of the numbers does not affect the result.	$(3 + 4) + 6 = 3 + (4 + 6)$	$(3 \cdot 4)5 = 3(4 \cdot 5)$

Example 1 Simplify: **a.** $75 + 13 + 25 + 47$ **b.** $4 \cdot 7 \cdot 25 \cdot 3$

Solution Regrouping makes mental math easier.

a. $75 + 13 + 25 + 47 = (75 + 25) + (13 + 47)$ Regroup the terms.
$= 100 + 60$ Simplify within the parentheses. Add.
$= 160$

b. $4 \cdot 7 \cdot 25 \cdot 3 = (4 \cdot 25)(7 \cdot 3)$ Regroup the factors.
$= 100 \cdot 21$ Simplify within the parentheses.
$= 2100$ Multiply.

Example 2 Simplify $1\frac{1}{3} + 16\frac{4}{5} + 2\frac{2}{3} + 3\frac{1}{5}$.

Solution Regroup the fractions. Simplify within the parentheses. Add.

$$1\frac{1}{3} + 16\frac{4}{5} + 2\frac{2}{3} + 3\frac{1}{5} = \left(1\frac{1}{3} + 2\frac{2}{3}\right) + \left(16\frac{4}{5} + 3\frac{1}{5}\right)$$
$$= 4 + 20$$
$$= 24$$

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2–1 Basic Assumptions *(continued)*

Example 3 Simplify $0.8 + 3.7 + 0.2 + 5.3$.

Solution Regroup the decimals. Simplify within the parentheses. Add.

$$0.8 + 3.7 + 0.2 + 5.3 = (0.8 + 0.2) + (3.7 + 5.3)$$
$$= 1 + 9$$
$$= 10$$

Simplify.

1. $125 + 42 + 75 + 28$
2. $507 + 36 + 43 + 14$
3. $2 \cdot 18 \cdot 5 \cdot 4$
4. $40 \cdot 3 \cdot 4 \cdot 20$
5. $50 \cdot 27 \cdot 4 \cdot 2$
6. $4 \cdot 15 \cdot 25 \cdot 3$
7. $3\frac{1}{2} + 5\frac{2}{3} + 2\frac{1}{2} + \frac{1}{3}$
8. $7\frac{2}{3} + 4\frac{3}{5} + 2\frac{1}{3} + \frac{12}{5}$
9. $0.2 + 3.9 + 2.8 + 0.1$
10. $0.6 + 5.2 + 0.4 + 3.8$
11. $2.85 + 3.75 + 1.15 + 9.25$
12. $3.25 + 1.95 + 8.75 + 11.05$

Example 4 Simplify: **a.** $6 + 8m + 4 + 7n$ **b.** $(3w)(2x)(4y)(5z)$

Solution

a. $6 + 8m + 4 + 7n = 8m + 7n + (6 + 4)$ Regroup the terms.
$= 8m + 7n + 10$ Simplify.

b. $(3w)(2x)(4y)(5z) = (3 \cdot 2 \cdot 4 \cdot 5)(wxyz)$ Regroup the factors.
$= 120wxyz$ Simplify.

Simplify.

13. $2 + 5y + 8$
14. $9 + 5z + 11$
15. $4 + 3x + 5$
16. $3 + 2w + 4$
17. $3(20a)$
18. $4(5n)$
19. $(5x)(6y)$
20. $(8m)(5n)$
21. $(6x)(y)(4z)$
22. $(2p)(3q)(5r)$
23. $(3a)(7b)(c)$
24. $(e)(6f)(2g)$
25. $a + 2 + b + 5$
26. $9 + x + y + 3$
27. $3p + 4 + 2q + 6$
28. $7m + 1 + 5n + 4$
29. $4 + 6x + 2 + 3y$
30. $6p + 3 + 2q + 37$
31. $(5a)(4b)(25c)(8d)$
32. $(4w)(2x)(5y)(5z)$

Mixed Review Exercises

Evaluate if $a = 2$, $x = 4$, $y = 6$, and $z = 3$.

1. $\frac{3x - a}{a + z}$
2. $4z(y - a)$
3. $\frac{2a + x}{3z - (y + 2)}$

Simplify.

4. $|-3| + |-5|$
5. $\left|-\frac{1}{6}\right| + 0$
6. $|-3.2| + |3.2|$
7. $|8| - |-8|$
8. $|-4| - |-2|$
9. $\left|-\frac{5}{7}\right| - \left|\frac{3}{7}\right|$